Spin Relaxation Mechanisms

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Highlight the physics of some spin relaxation mechanisms

There are some common spin relaxation mechanisms:

- Elliot-Yafet
- Dyakanov-Perel

I. SYMMETRY

A. Time-Reversal Symmetry

The time reversal symmetry implies that the system remains unchanged upon application of time-reversal operator. For an eigenstate characterized by quantum numbers $(\boldsymbol{k}, \boldsymbol{\sigma})$, time-reversal invariance means

$$\varepsilon_{\boldsymbol{k},\boldsymbol{\sigma}} = \varepsilon_{-\boldsymbol{k},-\boldsymbol{\sigma}} \tag{1}$$

which means that for every state characterized by quantum numbers $(\boldsymbol{k}, \boldsymbol{\sigma})$, there is a degenerate state $(-\boldsymbol{k}, -\boldsymbol{\sigma})$ also referred as ***Kramers Degeneracy***. The time-reversal operator $\hat{K}$ is expressed as

$$\hat{K} = -i\sigma_y \hat{C} \tag{2}$$

where σ_y is the y-Pauli matrix and $\hat{C}$ is the conjugation operator. Effects of $\hat{K}$:

$$\begin{aligned} &\boldsymbol{k} \to -\boldsymbol{k} \\ &|\uparrow\rangle \to |\downarrow\rangle \\ &|\downarrow\rangle \to -|\uparrow\rangle \end{aligned} \tag{3}$$

Note: For a particle with spin J, the representation for the operator is

$$\hat{K} = \exp\left(-i\pi J_y/\hbar\right)\hat{C} \tag{4}$$

If spin 1/2: $J_y = \hbar\sigma_y/2$ which implies $\exp\left(-i\pi J_y/\hbar\right) = -i\sigma_y$. This uses the identity (for given unit vector $\boldsymbol{v}$)

$$\begin{aligned} \exp\left(i\theta\boldsymbol{v}\cdot\boldsymbol{\sigma}\right) &= \sum_{k=0}^{\infty} \frac{1}{k\,!}\left(i\theta\boldsymbol{v}\cdot\boldsymbol{\sigma}\right)^k \\ &= \sum_{k=0}^{\infty} \frac{1}{(2k)\,!}\left(i\theta\boldsymbol{v}\cdot\boldsymbol{\sigma}\right)^{2k} + \sum_{k=0}^{\infty} \frac{1}{(2k+1)\,!}\left(i\theta\boldsymbol{v}\cdot\boldsymbol{\sigma}\right)^{2k+1} \\ &= \sum_{k=0}^{\infty} \frac{(-1)^k}{(2k)\,!}\theta^{2k}\left(\boldsymbol{v}\cdot\boldsymbol{\sigma}\right)^{2k} + \sum_{k=0}^{\infty} \frac{i(-1)^k}{(2k+1)\,!}\theta^{2k+1}\left(\boldsymbol{v}\cdot\boldsymbol{\sigma}\right)^{2k+1} \end{aligned} \tag{5}$$

Since $\left(\boldsymbol{v}\cdot\boldsymbol{\sigma}\right)^{2k} = I$ given $\boldsymbol{v}$ is a unit vector. Therefore

$$\begin{aligned} \exp\left(i\theta\boldsymbol{v}\cdot\boldsymbol{\sigma}\right) &= \sum_{k=0}^{\infty} \frac{(-1)^k}{(2k)\,!}\theta^{2k} I + \sum_{k=0}^{\infty} \frac{i(-1)^k}{(2k+1)\,!}\theta^{2k+1}\left(\boldsymbol{v}\cdot\boldsymbol{\sigma}\right) \\ &= \cos(\theta) I + i\sin(\theta)\boldsymbol{v}\cdot\boldsymbol{\sigma} \end{aligned} \tag{6}$$

B. Inversion Symmetry

The spatial inversion symmetry implies that the system remains unchanged upon application of inversion operator $\hat{I}$. For an eigenstate characterized by quantum numbers $(\boldsymbol{k}, \boldsymbol{\sigma})$, time-reversal invariance means

$$\varepsilon_{\boldsymbol{k},\boldsymbol{\sigma}} = \varepsilon_{-\boldsymbol{k},\boldsymbol{\sigma}} \tag{7}$$

which means that for every state characterized by quantum numbers $(\boldsymbol{k}, \boldsymbol{\sigma})$, there is a degenerate state $(-\boldsymbol{k}, \boldsymbol{\sigma})$. Effects of $\hat{I}$:

$$\begin{aligned} \boldsymbol{k} &\to -\boldsymbol{k} \\ \boldsymbol{r} &\to -\boldsymbol{r} \end{aligned} \tag{8}$$

C. Time-Reversal and Inversion Symmetry

When the system has both inversion and time-reversal symmetry

$$\varepsilon_{\boldsymbol{k},\boldsymbol{\sigma}} = \varepsilon_{-\boldsymbol{k},-\boldsymbol{\sigma}} = \varepsilon_{\boldsymbol{k},-\boldsymbol{\sigma}} = \varepsilon_{-\boldsymbol{k},\boldsymbol{\sigma}} \tag{9}$$

which implies that for every state characterized by quantum numbers $(\boldsymbol{k}, \boldsymbol{\sigma})$, there is a degenerate state $(\boldsymbol{k}, -\boldsymbol{\sigma})$.

II. ELLIOT-YAFET MECHANISM

This mechanism is based on scattering in presence of spin-orbit coupling. We know that the presence of spin-orbit leads to mixing the the spin up/down eigenstates. Thus a general eigenfunction can be written as

$$\Psi_{\boldsymbol{k},n,+1/2}(\boldsymbol{r}) = \left[a_{\boldsymbol{k},n}(\boldsymbol{r})|\uparrow\rangle + b_{\boldsymbol{k},n}(\boldsymbol{r})|\downarrow\rangle\right] e^{i\boldsymbol{k}\cdot\boldsymbol{r}} \tag{10}$$

Spin-Orbit Interaction preserves time-reversal symmetry, thus the time-reversed wavefunction has the same energy

$$\hat{K}\Psi_{\boldsymbol{k},n,+1/2}(\boldsymbol{r}) = \left[a^*_{-\boldsymbol{k},n}(\boldsymbol{r})|\downarrow\rangle - b^*_{-\boldsymbol{k},n}(\boldsymbol{r})|\uparrow\rangle\right] e^{i\boldsymbol{k}\cdot\boldsymbol{r}} \equiv \Psi_{-\boldsymbol{k},n,-1/2}(\boldsymbol{r}) \tag{11}$$

Therefore

$$\Psi_{\boldsymbol{k},n,-1/2}(\boldsymbol{r}) = \left[a^*_{\boldsymbol{k},n}(\boldsymbol{r})|\downarrow\rangle - b^*_{\boldsymbol{k},n}(\boldsymbol{r})|\uparrow\rangle\right] e^{-i\boldsymbol{k}\cdot\boldsymbol{r}} \tag{12}$$

Now if a scattering event happens which does not act in spin space e.g. impurity or phonons, the momenta of the electron changes from $\boldsymbol{k} \to \boldsymbol{k'}$. Thus the scattering probability to preserve/flip spin is

$$\begin{aligned} P_{|\uparrow\rangle\to|\uparrow\rangle} &\propto |M_{\boldsymbol{k},\boldsymbol{k'}}|^2 |a^*_{\boldsymbol{k'},n} a_{\boldsymbol{k},n}|^2 \\ P_{|\uparrow\rangle\to|\downarrow\rangle} &\propto |M_{\boldsymbol{k},\boldsymbol{k'}}|^2 |b^*_{\boldsymbol{k'},n} a_{\boldsymbol{k},n}|^2 \end{aligned} \tag{13}$$

This is the basic principle of Elliot-Yafet Spin Relaxation where non-spin-flip scattering events can lead to spin relaxation.
Physically, we expect scattering rate τ^{-1} increases with increase in temperature T. Thus we can conclude that with increase in temperature, the spin relaxation rate via Elliot-Yafet mechanism also increases.

$$\tau^{-1}_{EY} \propto \tau^{-1} \propto T \tag{14}$$

III. DYAKANOV-PEREL MECHANISM

If the system has both time-reversal and inversion symmetry

$$\varepsilon_{\boldsymbol{k},\boldsymbol{\sigma}} = \varepsilon_{-\boldsymbol{k},\boldsymbol{\sigma}} = \varepsilon_{\boldsymbol{k},-\boldsymbol{\sigma}} \tag{15}$$

which implies that for a given $\boldsymbol{k}$, there are two degenerate wavefunctions corresponding to spin up and down. Now if the system lacks inversion (in semiconductor nanostructures), only time-reversal symmetry holds

$$\varepsilon_{\boldsymbol{k},\boldsymbol{\sigma}} = \varepsilon_{-\boldsymbol{k},-\boldsymbol{\sigma}} \tag{16}$$

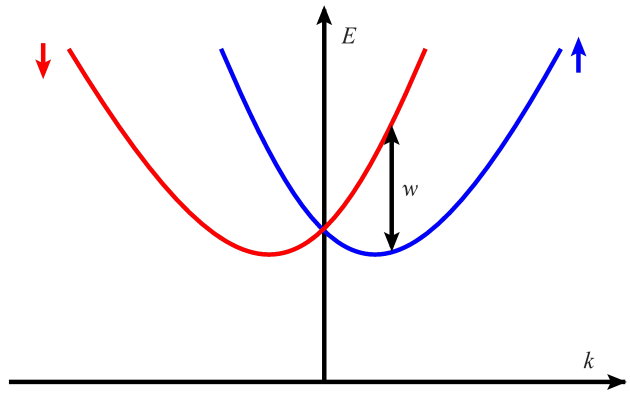

FIG. 1. No inversion symmetry: model energy dispersion

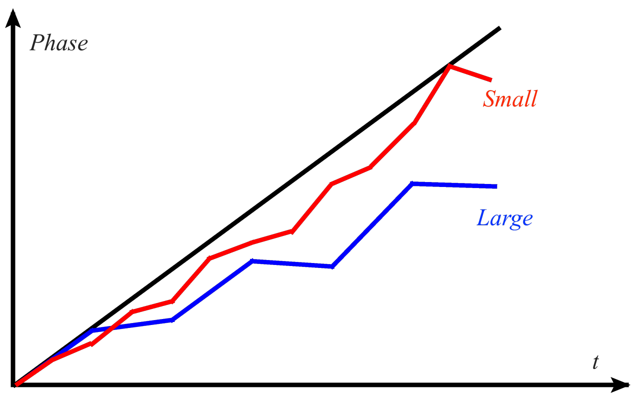

FIG. 2. Dyakanov-Perel phase relaxation. Small momentum relaxation time (red) and large momentum relaxation time (blue). Clearly the large τ deviates more and leads to more spin relaxation compared to small τ. This is counter intuitive and opposite to Elliot-Yafet.

This means that for a given wavevector $\boldsymbol{k}$, spin up and down are not degenerate. Thus we can think of spin-orbit coupling to be an effective $\boldsymbol{k}$-dependent magnetic field.

$$H_{SO} = \frac{1}{2}\boldsymbol{\Omega}(\boldsymbol{k}) \cdot \boldsymbol{\sigma} \tag{17}$$

Thus in presence of any momentum scattering, the electron sees different effective fields and precesses. This random-walk like precession leads to spin relaxation.

In presence of a constant magnetic field, the phase of an electron increases linearly with time with slope set by the Larmor frequency. For substantial phase randomization, we need larger τ, else the deviation of phase from the constant magnetic field line is not significant. Thus

$$\tau_{DP}^{-1} \propto \tau \propto 1/T \tag{18}$$