Neel Skyrmion: Energetics

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Let us consider a exchange coupled magnetic thin film (thickness L) with an out-of-plane uniaxial easy axis, interfacial DMI (caused by a heavy metal substrate), and an externally applied magnetic field along the out-of-plane normal. The energy describing this magnet is

$$E_{\rm total} = E_{\rm ex} + E_{\rm uni} + E_{\rm dmi} + E_{\rm z} \tag{1}$$

The magnetization unit vector can be described in terms of angles $\Theta(\vec{r})$ and $\Phi(\vec{r}) = \nu\phi + \gamma$ which are functions of in-plane spatial coordinates ($\vec{r} = \{r\cos\phi, r\sin\phi\}$) such that

$$\begin{aligned}
E_{\rm z} &= M_s B L \int dS\,[1 - m_z] \\
&= 2\pi M_s B L \int r dr\,[1 - \cos\Theta] \\
E_{\rm uni} &= M_s K_u L \int dS\,[1 - m_z^2] \\
&= 2\pi M_s K_u L \int r dr\,\sin^2\Theta \\
E_{\rm ex} &= AL \int dS\,[(\vec{\nabla} m_x)^2 + (\vec{\nabla} m_y)^2 + (\vec{\nabla} m_z)^2] \\
&= 2\pi AL \int r dr \left[\left(\frac{d\Theta}{dr}\right)^2 + \frac{\sin^2\Theta}{r^2}\right] \\
E_{\rm dmi} &= D_{\rm dmi} L \int dS\,[m_z \vec{\nabla}\cdot\vec{m} - \vec{m}\cdot\vec{\nabla} m_z] \\
&= 2\pi D_{\rm dmi} L \cos\gamma \int r dr \left[\frac{d\Theta}{dr} + \frac{\sin 2\Theta}{2r}\right]
\end{aligned} \tag{2}$$

where ν is the vorticity (winding number; set to be 1) and γ defines the skyrmion type ($\gamma = 0/\pi$- Neel; $\gamma = \pm\pi/2$-Bloch). Defining polarity as

$$p = \frac{m_z(0) - m_z(\infty)}{2} \tag{3}$$

and considering the situation

- where $p = 1$ meaning magnetization is along +z at the skyrmion core center and along -z outside the skyrmion core, we can see that the angle Θ increases monotonically as one moves radially outward from the core center implying $d\Theta/dr > 0$. On the other hand the integral of $\sin\Theta\cos\Theta$ is very small as the product has opposite signs on either side of the Skyrmion wall. Assuming that the interfacial DMI energy $D_{\rm dmi} > 0$, it can be seen that $\gamma = \pi$ will lower the energy of the system. Thus, we would have a Neel Skyrmion with $\gamma = \pi$.
- where $p = -1$ meaning magnetization is along -z at the skyrmion core center and along +z outside the skyrmion core, we can see that the angle Θ decreases monotonically as one moves radially outward from the core center implying $d\Theta/dr < 0$. On the other hand the integral of $\sin\Theta\cos\Theta$ is very small as the product has opposite signs on either side of the Skyrmion wall. Assuming that the interfacial DMI energy $D_{\rm dmi} > 0$, it can be seen that $\gamma = 0$ will lower the energy of the system. Thus, we would have a Neel Skyrmion with $\gamma = 0$.

An ansatz for the Neel Skyrmion (of radius R_s and wall width w_s) with polarity 1 is

$$m_z = \frac{\sinh(R_s/w_s) - \sinh(r/w_s)}{\sinh(R_s/w_s) + \sinh(r/w_s)} \tag{4}$$

equivalent to

$$\Theta = 2\tan^{-1}\left[\frac{\sinh(R_s/w_s)}{\sinh(r/w_s)}\right] \tag{5}$$

Following which

$$
\begin{aligned}
m_x &= \sqrt{1-m_z^2}\cos[\nu\phi+\gamma] \\
m_y &= \sqrt{1-m_z^2}\sin[\nu\phi+\gamma]
\end{aligned} \tag{6}
$$

Note that the ansatz can be simply generalized for Bloch skyrmions (However, we might need free energy terms corresponding to Bulk DMI and dipole-dipole interaction for understanding the stability of the soliton).