# Skyrmions: Emergent Fields

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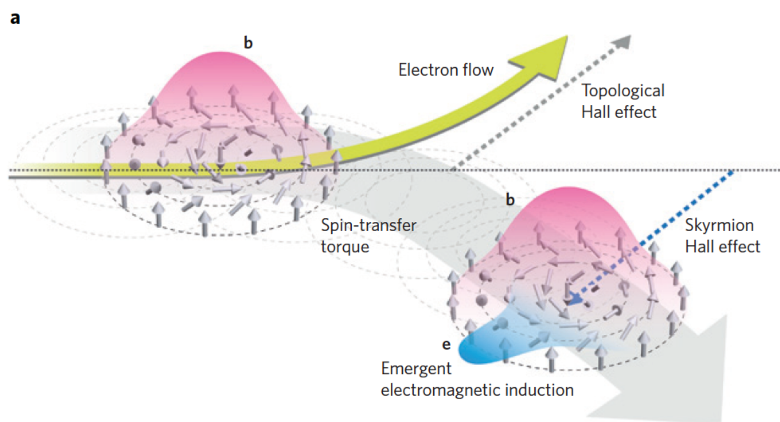


FIG. 1. Schematic for phenomenon arising from non-trivial topology of skyrmions. Ref: 'Topological properties and dynamics of magnetic skyrmions'- Nagaosa and Tokura, Nature Nanotechnology 8, 899 (2013)

We understand the non-trivial topology of skyrmion magnetic textures. Here, we would like to consider the emergent EM field that a propagating conduction electron experiences when traversing through such a topological texture. To develop a simplistic understanding, we assume that there is a strong Hund's exchange coupling between the electron and local magnetization. As a consequence of this strong coupling, the spin part of the wavefunction of the conduction electron is oriented in parallel to the local magnetization, such that

$$|\chi(\vec{r})\rangle = \begin{pmatrix} \cos\dfrac{\Theta}{2} \\ e^{i\Phi}\sin\dfrac{\Theta}{2} \end{pmatrix} \tag{1}$$

where $\Theta$ and $\Phi$ describe the orientation of the local magnetization. The motion of a electron in a lattice is described in terms of a hopping matrix element (in tight binding model). Such a hopping as a result of lattice translations along different directions $\alpha$ can be computed

$$t_\alpha(\vec{r}) = t\langle\chi(\vec{r})|\chi(\vec{r}+c\hat{\eta}_\alpha)\rangle \tag{2}$$

where $t$ is the bare hopping, $c$ is the lattice constant. A simple series expansion

$$|\chi(\vec{r}+c\hat{\eta}_\alpha)\rangle \approx |\chi(\vec{r})\rangle + c\,\partial_\alpha|\chi(\vec{r})\rangle \tag{3}$$

Consequently,

$$\begin{aligned} t_\alpha(\vec{r}) &= t[1 + c\,\langle\chi(\vec{r})|\partial_\alpha\chi(\vec{r})\rangle] \\ &= te^{ic\,a_\alpha(\vec{r})} \end{aligned} \tag{4}$$

where $a_\alpha(\vec{r}) = -i\langle\chi(\vec{r})|\partial_\alpha\chi(\vec{r})\rangle$ is the effective vector potential experienced by the electron. Utilizing

$$|\partial_\alpha\chi(\vec{r})\rangle = \begin{pmatrix} -\dfrac{1}{2}\sin\dfrac{\Theta}{2}\partial_\alpha\Theta \\ i\partial_\alpha\Phi e^{i\Phi}\sin\dfrac{\Theta}{2} + \dfrac{1}{2}e^{i\Phi}\cos\dfrac{\Theta}{2}\partial_\alpha\Theta \end{pmatrix} \tag{5}$$

implies

$$a_\alpha(\vec{r}) = \frac{1}{2}\partial_\alpha\Phi[1-\cos\Theta] \tag{6}$$

We can look at the out-of-plane magnetic field originating from the emergent vector potential

$$b_z = \partial_x a_y - \partial_y a_x = \frac{1}{2}\vec{m} \cdot (\partial_x \vec{m} \times \partial_y \vec{m}) \tag{7}$$

Consequently, the magnetic flux thought the x-y plane corresponding to the magnetic field is related to the topological skyrmion charge

$$\Phi_B = 2\pi\, Q_{\rm sk} \tag{8}$$

A consequence of this emergent magnetic field leads to a Hall effect referred to as 'Topological Hall Effect'. On the other hand, electron charge current applies a spin transfer torque (STT) which due to the presence of Gyrovector causes the skyrmion to move transverse to the direction of electric current. This is referred to as the 'Skyrmion Hall Effect'.

To determine the emergent electric field in addition to the magnetic field described above, we can alternatively look at the Schrodinger equation for the electron,

$$i\partial_t \psi(\vec{r}, t) = \left[-\frac{\nabla^2}{2m} - J\vec{\sigma} \cdot \vec{m}\right] \psi(\vec{r}, t) \tag{9}$$

The problem gets easier to tackle if the spin quantization axis aligns with the local magnetization orientation. To do so, we can perform a unitary transformation corresponding to rotation by an angle $\Theta(\vec{r}, t)$ about an axis that is normal to both the z-axis and the local magnetization orientation $\hat{n} = \hat{z} \times \vec{m}$,

$$U(\vec{r}, t) = \exp\left(-i\frac{\Theta(\vec{r}, t)}{2}\vec{\sigma} \cdot \hat{n}(\vec{r}, t)\right) \tag{10}$$

We can now rewrite the Schrodinger equation in terms of wavefunction $\phi(\vec{r}, t)$ such that $\psi(\vec{r}, t) = U(\vec{r}, t)\phi(\vec{r}, t)$,

$$i\partial_t \phi(\vec{r}, t) + [iU^\dagger \partial_t U]\phi(\vec{r}, t) = \left[\frac{(-i\nabla - iU^\dagger \nabla U)^2}{2m} - J\sigma_z\right] \phi(\vec{r}, t) \tag{11}$$

This is similar to the Schrodinger equation of a charged particle in presence of both scalar and vector potential,

$$i\partial_t \phi(\vec{r}, t) - q_e V(\vec{r}, t)\phi(\vec{r}, t) = \left[\frac{(-i\nabla - q_e \vec{A}(\vec{r}, t))^2}{2m} - J\sigma_z\right] \phi(\vec{r}, t) \tag{12}$$

Thus, the emergent potentials are defined as

$$V(\vec{r}, t) = -\frac{i}{q_e} U^\dagger \partial_t U; \quad \vec{A}(\vec{r}, t) = -\frac{i}{q_e} U^\dagger \nabla U \tag{13}$$

The emergent charge $q_e = \pm 1/2$ depending on the upper/lower band of the charge. One can now determine the emergent fields from the potentials,

$$\begin{aligned} B_i &= \epsilon_{ijk}\partial_j A_k = \frac{\epsilon_{ijk}}{2}\vec{m} \cdot (\partial_j \vec{m} \times \partial_k \vec{m}) \\ E_i &= -\partial_i V - \partial_t A_i = \vec{m} \cdot (\partial_i \vec{m} \times \partial_t \vec{m}) \end{aligned} \tag{14}$$