Plasmon Pole Approximation

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The objective is to discuss the plasmon pole approximation for the dielectric constant.

From simple equation of motion approach of electrons in a DC electric field, we know the dielectric constant to be

$$\varepsilon(q=0,\omega) = 1 - \frac{\omega_p^2}{\omega^2} \tag{1}$$

which explains the reflection of light from metals when the frequency of light is less than the plasma frequency ω_p. The inverse dielectric constant can thus be expressed as

$$\begin{aligned} \frac{1}{\varepsilon(q\to 0,\omega)} &= \frac{\omega^2}{(\omega+i\delta)^2 - \omega_p^2} \\ &= 1 + \frac{\omega_p^2}{(\omega+i\delta)^2 - \omega_p^2} \end{aligned} \tag{2}$$

where δ is a infinitesimally small. The above expression has a single pole at the plasma frequency. The plasmon-pole approximation is used to construct the full dielectric function $\varepsilon(q,\omega)$ which replaces the continuum of poles in the Lindhard function by one effective plasmon-pole ω_q. Therefore

$$\frac{1}{\varepsilon(q,\omega)} = 1 + \frac{\omega_p^2}{(\omega+i\delta)^2 - \omega_q^2} \tag{3}$$

The plasmon-pole ω_q is set by the sum rules that the dielectric function must satisfy. From the Kramers-Kronig relation ($\varepsilon = \varepsilon' + i\varepsilon''$)

$$\varepsilon'(q,\omega) = 1 + \frac{2}{\pi}\mathcal{P}\int_0^\infty d\omega' \, \frac{\omega' \, \varepsilon''(q,\omega')}{\omega'^2 - \omega^2} \tag{4}$$

In the long-wavelength static limit (q-very small and $\omega = 0$)

$$\varepsilon'(q,\omega=0) - 1 = \frac{\kappa^2}{q^2} = \frac{2}{\pi}\lim_{q\to 0}\int_0^\infty d\omega' \, \frac{\varepsilon''(q,\omega')}{\omega'} \tag{5}$$

where κ is the Thomas-Fermi screening wavevector. Therefore

$$\begin{aligned} \varepsilon(q,\omega) &= \frac{\omega^2 - \omega_q^2}{(\omega+i\delta)^2 - (\omega_q^2 - \omega_p^2)} \\ &= \frac{\omega^2 - \omega_q^2}{(\omega+i\delta-\Omega_q)(\omega+i\delta+\Omega_q)} \end{aligned} \tag{6}$$

where $\Omega_q^2 = \omega_q^2 - \omega_p^2$. We can read off the imaginary part of the dielectric constant

$$\begin{aligned} \varepsilon''(q,\omega) &= \frac{\omega^2 - \omega_q^2}{2\Omega_q}\text{Im}\left[\frac{1}{\omega+i\delta-\Omega_q} - \frac{1}{\omega+i\delta+\Omega_q}\right] \\ &= \frac{\omega^2 - \omega_q^2}{2\Omega_q}\left[-\pi\delta(\omega-\Omega_q) + \pi\delta(\omega+\Omega_q)\right] \end{aligned} \tag{7}$$

This implies

$$\begin{aligned} \frac{\kappa^2}{q^2} &= \frac{2}{\pi}\lim_{q\to 0}\frac{\pi}{2}\int_0^\infty d\omega' \, \frac{\omega^2 - \omega_q^2}{\Omega_q}\frac{\delta(\omega-\Omega_q)}{\omega'} \\ &= -\frac{\Omega_q^2 - \omega_q^2}{\Omega_q^2} = \frac{\omega_p^2}{\Omega_q^2} \end{aligned} \tag{8}$$

Hence

$$\begin{aligned}\Omega_q^2 &= \omega_p^2 \frac{q^2}{\kappa^2} \\ \Rightarrow \omega_q^2 &= \omega_p^2 \left[1 + \frac{q^2}{\kappa^2}\right]\end{aligned} \tag{9}$$

Thus within the plasmon-pole approximation, the dielectric function is approximated as

$$\frac{1}{\varepsilon(q,\omega)} = 1 + \frac{\omega_p^2}{(\omega + i\delta)^2 - \omega_q^2} \tag{10}$$

Lundquist added a correction to simulate the contribution of pair continuum

$$\omega_q^2 = \omega_p^2 \left[1 + \frac{q^2}{\kappa^2}\right] + \nu_q^2 = \omega_q^2 = \omega_p^2 \left[1 + \frac{q^2}{\kappa^2}\right] + Cq^4 \tag{11}$$

The above expressions are valid for 3D.

For 2D:

$$\omega_q^2 = \omega_p^2 \left[1 + \frac{q}{\kappa}\right] + Cq^2 \tag{12}$$