Mermin Wagner Hohenberg Theorem

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(Dated: February 28, 2018)

Discuss the Mermin Wagner Hohenberg Theorem

There is a celebrated theorem in equilibrium statistical mechanics, the Mermin-Wagner-Hohenberg-Coleman theorem, that essentially tells us that a continuous symmetry cannot be broken spontaneously at any finite temperature in dimensions two or lower. This is because the goldstone modes generated upon breaking breaking a continuous symmetry have strong fluctuations in d=1,2 leading to the symmetry being restored at long distances (for T>0).

Goldstone Theorem: Spontaneously breaking a continuous symmetry implies the appearance of a massless mode ($\omega = ck$) in the excitation spectrum of the system.

The proof is somewhat involved, thus we consider an alternate physical argument. The theorem states that there cannot exist a long range order in 2D at any finite temperature. Consider a magnetic system where the finite temperature introduces thermal fluctuations which are divergent and destroy ordering in dimensions two or lower.

Given the reduction in magnetization at finite temperature $\Delta M(T)$, the magnetization at any temperature is

$$M(T) = M(0) - \Delta M(T) \tag{1}$$

From statistical mechanics, we know that the finite temperature reduction in magnetization will depend on the density of state $N(E)$, occupation of the excitation mode,

$$\Delta M(T) \sim \int_0^\infty dE\, N(E) \frac{1}{\exp(E/k_B T) - 1} \tag{2}$$

For a general dispersion $E \sim k^n$ in d-dimension, the density of states per unit volume $N(E)$

$$N(E)dE = \frac{d^d k}{(2\pi)^d} = \frac{k^{d-1}}{(2\pi)^d} dk \tag{3}$$

$$N(E) = \frac{k^{d-1}}{(2\pi)^d} \frac{dk}{dE} \sim k^{d-n} \sim E^{(d-n)/n} \tag{4}$$

The dispersion of spin waves in Ferromagnets $\sim k^2$. Thus in 2D the density of states is a constant.

$$\Delta M(T) \sim \int_0^\infty dE\, \frac{1}{\exp(E/k_B T) - 1} \sim \int_0^\infty dx\, \frac{1}{\exp(x) - 1} \tag{5}$$

The integral clearly diverges at small x in a logarithmic manner. This means that the reduction in magnetization $\Delta M(T)$ diverges at finite temperature and causes a breakdown of magnetic order. This is due to the fact the finite temperature spin waves are infinitely easy to excite in 2D at finite temperature.

A. Effect of Anisotropy

NOTE: The above argument assumes isotropic interactions. If there is some anisotropy in the system (say an easy axis), the spin wave dispersion becomes gapped i.e. $E \sim A + Bk^2$. In this case

$$\Delta M(T) \sim \int_A^\infty dE\, \frac{1}{\exp(E/k_B T) - 1} \sim \int_a^\infty dx\, \frac{1}{\exp(x) - 1} \tag{6}$$

which does not diverge and thus magnetic order is stabilized by anisotropy.